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Variety of $SO(10)$ GUTs with Natural Doublet-Triplet Splitting via the Missing Partner Mechanism

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Abstract

We present a new class of unified $SO(10)$ models where the GUT symmetry breaking down to the standard model gauge group involves just one scale, in contrast to the conventional $SO(10)$ models which require two scales. Further, the models we discuss possess a natural doublet-triplet splitting via the missing partner mechanism without fine tuning. Such models involve $560+\overline{560}$ pair of heavy Higgs fields along with a set of light fields. The $560+\overline{560}$ are the simplest representations of $SO(10)$ besides the $126+\overline{126}$ which contain an excess of color triplets over $SU(2)_L$ doublets. We discuss several possibilities for realizing the missing partner mechanism within these schemes. With the $126+\overline{126}$ multiplets, three viable models are found with additional fields belonging to $\{210+2\times 10+120\}$, $\{45+10+120\}$, or $\{210+16+\overline{16}+10+120\}$. With the $560+\overline{560}$, a unique possibility arises for the missing partner mechanism, with additional $\{2\times 10+320\}$ fields. These models are developed in some detail. It is shown that fully realistic fermion masses can arise in some cases, while others can be made realistic by addition of vector–like representations. Naturally large neutrino mixing angles, including sizable θ_{13}, can emerge in these models. The couplings of the $H_u(H_d)$ Higgs doublets of the MSSM which give masses to the up quarks (down quarks and leptons) are not necessarily equal at the grand unification scale and would lead to a new phenomenology at the low energy scales.

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1 Introduction

Gauge symmetry based on $SO(10)$ provides a framework for unifying the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge groups and for unifying quarks and leptons in a single 16–plet spinor representation [1]. Additionally, the 16–plet also contains a right–handed singlet state, which is needed to give mass to the neutrino via the seesaw mechanism. Supersymmetric $SO(10)$ models have the added attraction that they predict correctly the unification of gauge couplings, and solve the hierarchy problem by virtue of SUSY. However, SUSY $SO(10)$ models, as usually constructed, have two drawbacks, both related to the symmetry breaking sector. Typically three types of Higgs fields are needed, e.g., $16 + \overline{16}$ for rank reduction, 45 for breaking the symmetry down to the standard model symmetry, and a 10 for electroweak symmetry breaking. The above implies that two different mass scales are involved in breaking of the GUT symmetry, one to reduce the rank and the other to reduce the symmetry all the way to $SU(3)_C \times SU(2)_L \times U(1)_Y$. Second, one must do an extreme fine–tuning at the level of one part in 10^{14} to get the Higgs doublets of MSSM light, while rendering super–heavy masses to their color–triplet GUT partners. We have previously investigated $SO(10)$ unified models wherein the first problem is addressed with some success. A single pair of $144 + \overline{144}$ dimensional vector-spinor Higgs multiplets can break the $SO(10)$ gauge symmetry in a single step all the way down to Standard Model (SM) gauge symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$ [2, 3, 4]. There we also exhibited the possibility to obtain from the same irreducible $144 + \overline{144}$ multiplets a pair of light Higgs doublets, necessary for the breaking of the electroweak symmetry. However, the second problem mentioned above, of extreme fine–tuning for making the Higgs doublets light, was not solved in that framework.

This doublet-triplet (DT) splitting problem is quite generic in grand unified models. One remedy proposed to solve the problem is the so called missing VEV mechanism [5] where the vacuum expectation value (VEV) of a 45 Higgs field which breaks the $SO(10)$ symmetry lies in the $(B - L)$–preserving direction, and generates masses for the Higgs triplets but not for the Higgs doublets from a 10–plet. This mechanism works in $SO(10)$ and has no analog in $SU(5)$. A second remedy is the missing partner mechanism, first discovered in the context of $SU(5)$ [6] and later investigated for $SO(10)$ [7]. In $SU(5)$, the missing partner mechanism requires the representations $50 + \overline{50} + 75$ (all heavy) in addition to the light $5 + \bar{5}$ of Higgs.[2] Now, it turns out the $50 + \overline{50}$ have one pair of Higgs triplets/anti-triplets and no Higgs doublets. Thus when they mix with the $5 + \bar{5}$, the Higgs triplets/anti-triplets in $5 + \bar{5}$ become heavy, while the Higgs doublets in $5 + \bar{5}$ remain light since there are no Higgs doublets in $50 + \overline{50}$ for them to pair with. The 75–plet Higgs is needed to induce the mixing of the color triplets from the $5 + \bar{5}$ and the $50 + \bar{5}0$. In Ref. [7] the missing partner mechanism was extended to $SO(10)$ by considering the set of heavy fields $126 + \overline{126} + 210$

[2]Heavy in this context means the presence of a GUT scale mass term for the field, while light means the absence of a GUT scale mass for the field. Components of a light field can become heavy via its mixing with a heavy field.

and a set of light fields. The lowest dimensional representations of $SO(10)$ that contain $50+\overline{50}$ of $SU(5)$ are the $126+\overline{126}$. The 210 is needed since it contains the 75 of $SU(5)$.

In this paper we first investigate systematically the missing partner $SO(10)$ models anchored by the $126+\overline{126}$. Here two new possibilities are found, in addition to the model proposed in Ref. [7]. Then we discuss a new class of $SO(10)$ models, where we solve the DT splitting problem via the missing partner mechanism, and simultaneously achieve the one-step GUT symmetry breaking of $SO(10)$. Here the missing partner mechanism is anchored by a pair of $560+\overline{560}$ Higgs fields, which also contain $50+\overline{50}$ under $SU(5)$. These fields are the next simplest representations containing an excess of color triplets over $SU(2)_L$ doublets beyond $126+\overline{126}$. In addition, the 560 contains in the $SU(5)\times U(1)$ decomposition $1(-5)+24(-5)+75(-5)$. When these fields acquire vacuum expectation values (VEVs), one ends up with the residual gauge group $SU(3)_C\times SU(2)_L\times U(1)_Y$. Thus $SO(10)$ gauge symmetry breaks down to the Standard Model in one step. An attractive feature of the $560+\overline{560}$ combination is that they contain $75+\overline{75}$–plet representations which is absent in the $126+\overline{126}$ pair, necessitating the inclusion of the 210–plet in that case. (75–plet of $SU(5)$ is self-dual, we use the notation 75 to denote $75(-5)$ and $\overline{75}$ to denote a $75(+5)$ with opposite $U(1)$ quantum numbers under $SO(10)\to SU(5)\times U(1)$.) The missing partner mechanism works in the case of 560 by its mixing with certain light fields. A unique possibility is found, with the light fields belonging to $\{2\times 10+320\}$ representations. A non-trivial constraint on such models for consistency is that all exotics (particle other than the Higgs doublets of the MSSM) from the $560+\overline{560}$ or $126+\overline{126}$ must acquire GUT scale masses, which is satisfied in the models presented here.

Some of the models developed lead to fully realistic fermion masses and mixings. For those which do not, we suggest a method of adding equal number of heavy and light fields to the Higgs spectrum outlined above, which leaves the missing partner mechanism intact. The MSSM fields H_u and H_d will now have an admixture of these newly added light field, which in turn makes the fermion masses fully realistic. In particular, we find that these models can naturally lead to large neutrino mixing angles, including a sizable θ_{13}.

The outline of the rest of the paper is as follows: In Sec. 2 we describe the essence of the missing partner mechanism in $SU(5)$. In Sec. 3 we discuss the missing partner mechanism anchored by $126+\overline{126}$. Here we identify three viable models which require no fine tuning. In Sec. 4 we discuss the newly proposed missing partner mechanism anchored by $560+\overline{560}$ Higgs fields. Since the models based on $560+\overline{560}$ are new and not familiar we discuss further details of these in Secs. 5-8. Thus in Sec. 5 we discuss the 560–plet as a constrained spinor-tensor multiplet, and exhibit explicitly its components for calculational purposes. In Sec. 6 we discuss the breaking of the GUT symmetry by $560+\overline{560}$ multiplet so that $SO(10)\to SU(3)_C\times SU(2)_L\times U(1)_Y$ using a single mass scale. Numerical solutions are presented to the six coupled minimization conditions involving the VEVs

of the sub-multiplets $1, 24, 75$ and $\bar{1}, \overline{24}, \overline{75}$. In Sec. 7 we compute the Higgs doublet and the Higgs triplet mass matrices in one model, i.e., the model with the Higgs fields $560 + \overline{560} + (2 \times 10 + 320)$. A brief discussion of how realistic fermion masses can arise in these models is presented in Sec. 8. Conclusions are given in Sec. 9. Some further calculational details on spontaneous breaking with $560 + \overline{560}$ multiplet are given in the Appendix.

2 Missing Partner Mechanism in $SU(5)$

Before discussing the missing partner mechanism in $SO(10)$ we briefly review the simpler case of missing partner mechanism in $SU(5)$. The simplest Higgs structure which breaks $SU(5)$ and contains Higgs doublets needed for the breaking of the electroweak symmetry consists of $5(H_2) + \bar{5}(H_1)$ and a 24–plet of Higgs with a superpotential of the type

$$\lambda \left[\frac{1}{2} M Tr(\Sigma^2) + \frac{1}{3} Tr(\Sigma^3) \right] + \lambda' H_{1x} (\Sigma^x_y + 2M' \delta^x_y) H_2^y. \tag{1}$$

Here, if the 24-plet develops a VEV of the form $\langle \Sigma \rangle = V \,\mathrm{diag}\,(2, 2, 2, -3, -3)$ arising from the minimization of the first two terms in Eq. (1), then $SU(5)$ breaks down to $SU(3)_C \times SU(2)_L \times U(1)$. However, the Higgs doublets and Higgs triplets/anti-triplets in $5+\bar{5}$ would gain super–heavy masses and a fine tuning of one part in 10^{14} is needed to make the Higgs doublets light, without also making the color triplets light.

To obtain Higgs doublets naturally light without fine tuning, one can utilize a different array of Higgs multiplets which are $5(H_2^i), \bar{5}(H_{1j}), 50^{ijk}_{lm}, \overline{50}^{ij}_{klm}, 75^{ij}_{kl}$, where the fields satisfy the following constraints: $50^{ijk}_{lk} = 0$, $50^{ijk}_{jk} = 0$, $\overline{50}^{in}_{lmn} = 0$, $\overline{50}^{mn}_{lmn} = 0$, $75^{ij}_{kj} = 0$, $75^{ij}_{ij} = 0$. We consider now a superpotential of the form

$$W_{Higgs} = W_0(75) + M\, 50 \cdot \overline{50} + \lambda_1 50 \cdot 75 \cdot \bar{5} + \lambda_2 \overline{50} \cdot 75 \cdot 5. \tag{2}$$

Here $W_0(75) = M_{75}\, 75^2 + \lambda\, 75^3$, which generates a VEV for the Standard Model singlet component of 75 and breaks the $SU(5)$ symmetry down to $SU(3)_C \times SU(2)_L \times U(1)_Y$. In the $SU(3)_C \times SU(2)_L \times U(1)$ decomposition the multiplets break as follows:

$$\begin{aligned}
5 &= (1,2)(3) + (3,1)(-2), \\
50 &= (1,1)(-12) + (3,1)(-2) + (\bar{3},2)(-7) + (\bar{6},3)(-2) + (6,1)(8) + (8,2)(3), \\
75 &= (1,1)(0) + (3,1)(10) + (3,2)(-5) + (\bar{3},1)(-10) + (\bar{3},2)(5) + \\
&\quad (\bar{6},2)(-5) + (6,2)(5) + (8,1)(0) + (8,3)(0)\,,
\end{aligned} \tag{3}$$

with the decomposition for the $\bar{5}$ and $\overline{50}$ obvious from Eq. (3). (The SM hypercharge $Y/2$ is $1/6$ times the charge quoted above.) The decomposition above shows that $5 + \bar{5}$ have one pair of Higgs doublets and one pair of Higgs triplets/anti-triplets while the $50 + \overline{50}$ have one pair of Higgs

triplets/anti-triplets and no Higgs doublets. Thus after the GUT symmetry breaks via the VEV of the 75–plet, the last three terms in the superpotential of Eq. (2) would generate masses for the Higgs triplets/anti-triplets while the Higgs doublets remain light because there are no Higgs doublets to pair up with in $50 + \overline{50}$. So this is the simplest way the missing partner mechanism works in $SU(5)$. Of course, as a GUT group $SO(10)$ is more desirable than $SU(5)$ and thus it is of interest to discuss the various possibilities to achieve the missing partner mechanism in $SO(10)$. The main aim of this paper is to explore these possibilities.

3 Missing Partner Mechanism in $SO(10)$ Anchored by $126 + \overline{126}$

In searching for the missing partner mechanism in $SO(10)$ we note once again that the mechanism works in $SU(5)$ because of the central fact that the $50+\overline{50}$ multiplets in $SU(5)$ contain an $SU(3)_C$ triplet/anti-triplet pair but no $SU(2)_L$ doublets. Thus to implement the mechanism at the level of $SO(10)$, a simple procedure would be to look for $SO(10)$ representations which contain the $SU(5)$ representations $50 + \overline{50}$. Now the lowest irreducible representation of $SO(10)$ where the $\overline{50}(50)$ appears is the $126(\overline{126})$ while the next one is $560(\overline{560})$ [8]. The case $126 + \overline{126}$ has been studied in [7]. Here we systematically investigate this case, and find three viable solutions without fine tuning, two of which are new. The 126 decomposes under $SU(5) \times U(1)_X$ as follows:

$$126 = 1(-10) + \bar{5}(-2) + 10(-6) + \overline{15}(6) + 45(2) + \overline{50}(-2). \tag{4}$$

Notice that the 126–plet does not contain the 75–plet of $SU(5)$, and thus to break the GUT symmetry we include a 210 representation which contains the 75–plet as is seen from the following decomposition under $SU(5) \times U(1)$,

$$210 = 1(0) + 5(-8) + \bar{5}(8) + 10(4) + \overline{10}(-4) + 24(0) + 40(-4) + \overline{40}(4) + 75(0). \tag{5}$$

Thus to achieve DT splitting in this case one needs a heavy sector consisting of $(126 + \overline{126} + 210)$. Since $126 + \overline{126}$ contain 2 doublet pairs and 3 triplet/anti-triplets pairs, and since 210 contains one doublet pair and one triplet/anti-triplets pair, one has a total of (3,4) (doublet, triplet/anti-triplets) pairs from the heavy sector. Suppose the light sector contains (4,4) (doublet, triplet/anti-triplets) pairs. If the light sector obtains mass only by mixing with the heavy sector, one will be left with just one pair of light Higgs doublets while all the triplets/anti-triplets will be heavy. We now discuss various ways of achieving this.

(i) Heavy $\{126 + \overline{126} + 210\}$ + Light $\{2 \times 10 + 120\}$ model:

Now, suppose we choose a set of $\{2 \times 10 + 1 \times 120\}$ light fields, along with $\{126 + \overline{126} + 210\}$ heavy fields. These light fields will have (4,4) (doublet, triplet/anti-triplets), as can be seen from

the following decompositions under $SU(5)\times U(1)$:

$$\begin{aligned} 10 &= 5(2)+\overline{5}(-2) \\ 120 &= 5(2)+\overline{5}(-2)+10(-6)+\overline{10}(6)+45(2)+\overline{45}(-2). \end{aligned} \tag{6}$$

Each of the 10's contains one (doublet, triplet/anti-triplets) pairs, while the 120 contains two such pairs, one pair from the $5+\overline{5}$ and one pair from the $45+\overline{45}$ fragments of Eq. (6). (The 45 of $SU(5)$ decomposes under $SU(3)_C\times SU(2)_L\times U(1)$ as $45=(1,2)(3)+(3,1)(-2)+(3,3)(-2)+(\overline{3},1)(8)+(\overline{3},2)(-7)+(\overline{6},1)(-2)+(8,2)(3)$, which shows its doublet/triplet content.) These light fields mix with the heavy $\{126+\overline{126}+210\}$ fields through the following set of couplings:

$$W_{DT}^{(i)} = 10_i\cdot 126\cdot 210 + 10_i\cdot\overline{126}\cdot 210 + 120\cdot 126\cdot 210 + 120\cdot\overline{126}\cdot 210\ . \tag{7}$$

Notice the absence of bare mass terms (or effective mass terms via the couplings $120^2\cdot 210$ and $120\cdot 10_i\cdot 210$) for the light fields in $\{2\times 10+120\}$. As per the counting listed above, these couplings would lead to 4 pairs of super–heavy color triplet/anti-triplets, and 3 pairs of super–heavy doublets, leaving one pair of light Higgs doublets, to be identified with H_u and H_d of MSSM. Further, all the remaining components of the light fields which do not enter in the DT splitting gain super–heavy masses. This point is quite non–trivial for the sub-multiplets from 120, but all of these fields pair up with sub-multiplets from the $126+\overline{126}$ or 210, as we now show.

In the $SU(5)\times U(1)$ decomposition the exotics in 120 are $10(-6),\overline{10}(6),45(2),\overline{45}(-2)$, see Eq. (6). The relevant superpotential that achieves natural DT splitting is given in Eq. (7). After spontaneous breaking the 75(0)–plet in 210 develops a VEV and generates mass for the exotics as follows.

$$\begin{aligned} &\langle(210,75(0))\rangle\cdot(126,10(-6))\cdot(120,(\overline{10}(6)), \qquad \langle(210,75(0))\rangle\cdot(\overline{126},\overline{10}(6))\cdot(120,10(-6)), \\ &\langle(210,75(0))\rangle\cdot(126,45(2))\cdot(120,\overline{45}(-2)), \qquad \langle(210,75(0))\rangle\cdot(\overline{126},\overline{45}(-2))\cdot(120,45(2))\ . \end{aligned} \tag{8}$$

Here the sub-multiplets under $SU(5)\times U(1)$ involved are explicitly indicated. We see that the 75(0) VEV of the 210–plet alone would give super-heavy masses to all exotics from the 120. There are additional contributions to the exotic masses from the VEVs of 24(0) and 1(0) fragments of the 210, which are analogous to the ones in Eq. (8). When the singlet VEVs $1(-10)$ from the 126 and $(1,+10)$ from the $\overline{126}$ are inserted in Eq. (7), additional mass terms would arise which mix the $\overline{10}(4)$ from the 120 with the 10(6) from the 210, and similarly the $10(-4)$ from the 120 with the $\overline{10}(-6)$ from the 210. Note that one combination of $10+\overline{10}$ pair under $SU(5)$ from the $210+126+\overline{126}$ are the Goldstone modes, but the exotic $10+\overline{10}$ components of 120 pair up with the orthogonal non–Goldstone components from the $210+126+\overline{126}$. Thus all the exotics in 120–plet are removed from the light spectrum.

The model just described is fully realistic. Symmetry breaking occurs consistently [9], there are no unwanted light states, and the masses and mixings of quarks and leptons are induced correctly.

The last fact follows from the two flavor symmetric sets of Yukawa couplings with the 10–plets of Higgs which contribute equally to down quark and charged lepton masses, an antisymmetric set of Yukawa coupling with the 120–plet which distinguishes down quarks and charged leptons, and the Yukawa couplings to the $\overline{126}$–plet which generates heavy masses for the right–handed neutrinos. In contrast, when the missing partner mechanism is employed in $SU(5)$, the degeneracy between the down quarks and charged leptons is not lifted. The simplest way out of the wrong mass predictions in this case would be to also add a vector–like pair of fermions in the $10 + \overline{10}$ representations, the components of which mix by different amounts with the down quarks and the charged leptons through couplings such as $10_i \overline{10}\ 75$ ($i = 1 - 3$ is the family index). Alternatively, one could resort to higher dimensional non–renormalizable operators. Generating neutrino masses via the seesaw mechanism would also require the introduction of a singlet fermion sector in $SU(5)$.

What about other possibilities for the light states in missing partner $SO(10)$ models with $\{126 + \overline{126} + 210\}$ heavy fields? Since the heavy sector has four pairs of color triplet/anti–triplet fields, it might appear that mixing four 10–plets with these heavy fields could also lead to the desired light Higgs doublets. However, this is not so. In this case, only two combinations of the four 10–plets will get involved in the light–heavy mixings analogous to Eq. (7), resulting in two 10–plets entirely becoming light. Similarly, adding pairs of $16 + \overline{16}$ which contain one pair of Higgs doublets and a pair of color triplet/anti–triplet does not work in any simple way. To emphasize this point, let us consider replacing some of the doublets and color triplets/anti-triplets of the light sector in the model described above by those from $16 + \overline{16}$. Under $SO(10) \to SU(5) \times U(1)$ we have $16 = 1(-5) + \overline{5}(3) + 10(-1)$. Thus $16 + \overline{16}$ contain one pair of doublets and one pair of color triplet/anti-triplets. They also contain an exotic $10 + \overline{10}$ pair under $SU(5)$ subgroup, which must be given large mass. Now, the heavy sector $\{126 + \overline{126} + 210\}$ contains two pairs of $10 + \overline{10}$, one of which pairs up with the gauge super–multiplet upon symmetry breaking. Thus, at most one $10 + \overline{10}$ exotic from the $16 + \overline{16}$ can be paired with those from the heavy fields. This implies that in the example with $\{2 \times 10 + 1 \times 120\}$ light fields, we cannot replace the 120 field – which contains two pairs of doublets and triplets – by 2 pairs of $16 + \overline{16}$. Replacing one light 10–plet by a $16 + \overline{16}$ might appear feasible, but in this case symmetry breaking will not occur consistently. The couplings $16 \cdot 16 \cdot \overline{126} + \overline{16} \cdot \overline{16} \cdot 126$ would provide the needed light–heavy mixing, if the SM singlet components of 16 and $\overline{16}$ acquire VEVs. Being the light field, one should not allow $16 \cdot \overline{16}$ mass term, nor the coupling $16 \cdot \overline{16}.210$. Setting the F–terms associated with the 16 and $\overline{16}$ fields to zero, one finds $\langle 16 \rangle \langle \overline{126} \rangle = 0$, and $\langle \overline{16} \rangle \langle 126 \rangle = 0$. One solution to these equations is $\langle 16 \rangle = 0, \langle \overline{16} \rangle = 0$, implying that the needed light–heavy mixing is not induced. If the other solution where $\langle 126 \rangle = 0, \langle \overline{126} \rangle = 0$ is chosen, F–term associated with the 126 and $\overline{126}$ would lead to $\langle 16 \rangle = 0, \langle \overline{16} \rangle = 0$, again leading to vanishing heavy–light mixing.

In spite of the various constraints that need to be satisfied, we have found two additional ways

of realizing the missing partner mechanism with the use of $126+\overline{126}$ pair that do not lead to light exotics.

(ii) Heavy $\{126+\overline{126}+45\}$ + Light $\{10+120\}$ model:

In this model the heavy Higgs sector consists of $\{126+\overline{126}+45\}$. Since the 45–plet does not contain color triplets or $SU(2)_L$ doublets ($45 = 1(0)+10(4)+\overline{10}(-4)+24(0)$ under $SU(5)\times U(1)$), the heavy sector will have two pairs of doublets and three pairs of color triplet/anti-triplets, all arising from the $126+\overline{126}$. The light sector is taken to be $\{10+120\}$ which contains three pairs of doublets and three pairs of triplets/anti-triplets. The relevant superpotential for doublet–triplet splitting is

$$W_{DT}^{(ii)} = 10\cdot 126\cdot 45^2 + 10\cdot\overline{126}\cdot 45^2 + 120\cdot 126\cdot 45 + 120\cdot\overline{126}\cdot 45\ . \tag{9}$$

The 45^2 in the first two terms of Eq. (9) act effectively as a 210–plet of model (i). DT splitting would then work in analogy with Eq. (7), although in the present case there is only a single 10–plet light state (as opposed to two light 10–plets in Eq. (7)). This difference arises because, unlike the heavy 210 which contains a pair of doublets and triplets, the 45–plet employed in Eq. (9) contains neither of these fields. Although this new model utilizes dimension four terms in Eq. (9) and in the superpotential for symmetry breaking (terms such as 45^4 are necessary for the $SO(10)$ symmetry to break down to the Standard Model symmetry), the particle content of the model is relatively simple, making this model attractive. The exotics from the 120–plet all acquire super–heavy masses by pairing with components of $126+\overline{126}$ or 45. When the SM singlet from the 24(0) of 45 acquires a VEV, the last two terms of Eq. (9) would generate the following mass terms.

$$\begin{aligned}
&\langle(45,24(0))\rangle\cdot(126,10(-6))\cdot(120,(\overline{10}(6)), \qquad \langle(45,24(0))\rangle\cdot(\overline{126},\overline{10}(6))\cdot(120,10(-6)),\\
&\langle(45,24(0))\rangle\cdot(126,45(2))\cdot(120,\overline{45}(-2)), \qquad \langle(45,24(0))\rangle\cdot(\overline{126},\overline{45}(-2))\cdot(120,45(2))\ .
\end{aligned} \tag{10}$$

Additionally, when the VEVs of $1(-6)$ of the 126 and the $1(6)$ of the $\overline{126}$ are inserted in Eq. (9), $(120,10(-6))\cdot(45,\overline{10}(-4))\cdot\big\langle(\overline{126},1(10))\big\rangle$ and $(120,\overline{10}(6))\cdot(45,10(4))\cdot\langle(126,1(-10))\rangle$ will be induced, providing additional mass corrections to the exotics from the 120. Thus we see that all fragments from the 120, except for the MSSM Higgs doublet components, have become massive.

The fermion mass matrices that arise from the symmetric Yukawa couplings of the 10–plet and the antisymmetric Yukawa couplings of the 120–plet do not lead to fully consistent charged fermion masses and mixings. A simple solution to fix this problem, without upsetting the DT mechanism is provided in Sec. 8. Adding equal number of light and heavy fields to the spectrum of the present model does not upset DT splitting mechanism, since the counting of doublets and triplets is unaltered. If a pair of $126+\overline{126}$ fields are added to the heavy and the light spectrum, H_u and H_d

of the MSSM would have components from the $\overline{126}$, which would correct the wrong mass relations.

(iii) Heavy $\{\mathbf{126}+\overline{\mathbf{126}}\}$ + Light $\{\mathbf{10}+\mathbf{120}\}$ model:

The last possibility for realizing the missing partner mechanism utilizing the $\{126+\overline{126}\}$ heavy fields is the one developed in Ref. [7], which we briefly summarize. Here one uses a $\{126+\overline{126}\}$ pair of heavy fields with the feature that these fields do not acquire VEVs. There are additional heavy Higgs fields belonging to $\{210+16+\overline{16}\}$ which acquire VEVs and break the $SO(10)$ gauge symmetry down to $SU(3)_C\times SU(2)_L\times U(1)_Y$. The doublets and color triplets from these $\{210+16+\overline{16}\}$ fields do not mix with the doublets and triplets from the heavy $126+\overline{126}$ fields, nor with those from the light sector. The light fields consist of $\{10+120\}$, which contain a total of three doublet pairs and three color triplet/anti-triplet pairs. When the light sector mixes with the heavy $\{126+\overline{126}\}$ fields, and not with the heavy $\{210+16+\overline{16}\}$ fields responsible for $SO(10)$ symmetry breaking, all three color triplet/anti-triplet pairs become massive by pairing with the three color triplets/anti-triplets from the $\{126+\overline{126}\}$. Only two of the doublet pairs from the $\{10+120\}$ become massive by mixing, since the $\{126+\overline{126}\}$ contain two pairs of doublets. Thus one pair of doublets becomes light. The relevant superpotential couplings are

$$W_{DT}^{(iii)} = 10\cdot 126\cdot 210 + 10\cdot\overline{126}\cdot 210 + 120\cdot 126\cdot 210 + 120\cdot\overline{126}\cdot 210\ . \tag{11}$$

Once the 75 plet of $SU(5)$ in the 210 of $SO(10)$ acquires a VEV, all exotics from the 120 would acquire masses, as in Eq. (8) of model (i). The main difference of this model, compared to Eq. (7) is that here the $\{126+\overline{126}\}$ heavy fields do not acquire VEVs, and as a result the light doublet count is different.

It was shown in Ref. [7] that there exists a $U(1)$ symmetry that forbids all unwanted couplings to sufficiently high order. This includes the absence of mass terms for the light fields. In this paper we simply use the supersymmetric non-renormalization theorem to set the light field masses (or effective masses) to small values.

4 Missing Partner Mechanism Anchored by $560+\overline{560}$

As mentioned in the introduction, the second smallest irreducible representation after $126(\overline{126})$ with the feature that it contains $\overline{50}(50)$ of $SU(5)$ needed for missing partner mechanism is $560(\overline{560})$. An additional nice feature of the $560(\overline{560})$ set is that it also contains the $75(\overline{75})$–plet representation. Unlike the case of $126+\overline{126}$ where a 210–plet was needed for this purpose, here there is no need for any other Higgs fields for symmetry breaking. We shall demonstrate how the $560+\overline{560}$ pair of Higgs breaks the $SO(10)$ symmetry down to $SU(3)_C\times SU(2)_L\times U(1)_Y$ in one step in Sec. 6. Here we study its attributes for realizing the missing partner mechanism in a simple and non-technical way, with more technical discussions given in Secs. 5 and 6. The 560 is a tensor–spinor representation,

denoted as $\Psi^\alpha_{\mu\nu}$, where $\alpha = 1-16$ is the spinor index, and $\mu,\nu = 1-10$ are the tensor indices. $\Psi^\alpha_{\mu\nu}$ obeys the following conditions: $\Gamma_\mu \Psi^\alpha_{\mu\nu} = 0$, and $\Psi^\alpha_{\mu\nu} = -\Psi^\alpha_{\nu\mu}$, where Γ_μ are the $SO(10)$ gamma matrices. With these conditions, $\Psi^\alpha_{\mu\nu}$ has 560 independent components. The $560 + \overline{560}$ multiplets will have to mix with a certain set of light fields. To see the various possibilities, let us note the decomposition of 560 under $SU(5) \times U(1)$ [8]:

$$
\begin{aligned}
560 &= 1(-5) + \bar{5}(3) + \overline{10}(-9) + 10(-1)_1 + 10(-1)_2 + 24(-5) + 40(-1) \\
&+ 45(7) + \overline{45}(3) + \overline{50}(3) + \overline{70}(3) + 75(-5) + 175(-1) \ .
\end{aligned}
\tag{12}
$$

The 560 contains one up–type Higgs doublet, three down–type doublets, one color triplet and four color anti–triplets. This follows from Eq. (12), but can also be inferred from the decomposition of 560 under $SU(2)_L \times SU(2)_R \times SU(4)_C$ given in Ref. [8]. Thus $560+\overline{560}$ contain four pairs of Higgs doublets and five pairs of color triplets/anti-triplets. If these fields mix with light fields containing five pairs of doublets and triplets, one pair of Higgs doublets from the light fields will remain light, to be identified as H_u and H_d of MSSM.

To see the various possibilities for the light fields, it is necessary to find out the decomposition of 560×560, which is not readily available [8]. We have evaluated this from the publicly available numerical program LiE [10]. Since we employ only one pair of $560 + \overline{560}$, it is important to see which components appear in the symmetric combination. The product rule, in the symmetric and antisymmetric combinations, are given by

$$
\begin{aligned}
(560 \times 560)_s &= 10 + 126_1 + 126_2 + \overline{126} + 320 + 210' + 1728_1 + 1728_2 + 2970_1 + 2970_2 \\
&+ 3696 + 4410 + 4950 + \overline{4950} + 10560 + 6930' + 36750 + 27720 + 46800 \ , \\
(560 \times 560)_a &= 120_1 + 120_2 + 320 + 1728_1 + 1728_2 + 2970 + 3696_1 + 3696_2 \\
&+ 4312_1 + 4312_2 + 10560 + 36750 + 34398 + 48114 \ .
\end{aligned}
\tag{13}
$$

(iv) Heavy $\{560 + \overline{560}\}$ + Light $\{2 \times 10 + 320\}$ model:

From Eq. (13), an essentially unique possibility for DT splitting via the missing partner mechanism emerges. This uses $\{2 \times 10 + 320\}$–plets as the light fields. The 320 contains three pairs of doublets and triplets, and with the two pairs of doublets and triplets from the two 10–plets, this choice becomes consistent with pairing up all five color triplets, while pairing only four sets of doublets with those from the $560 + \overline{560}$. The 320 multiplet is interesting and to our knowledge has not been utilized in model building in particle physics. The 320–plet can be taken to be a three index tensor which is anti-symmmetric in the first two indices with the totally anti-symmetric part (120) taken out and is traceless so that 450-120-10=320. Under $SU(5) \times U(1)$ it decomposes as

$$
320 = 5(2) + \bar{5}(-2) + 40(-6) + \overline{40}(6) + 45(2) + \overline{45}(-2) + 70(2) + \overline{70}(-2), \tag{14}
$$

which shows that it contains 3 doublet and 3 triplet/anti-triplet pairs.

We assume a superpotential of the form

$$\mathsf{W}' = M_{560}\ 560 \cdot \overline{560} + 560 \cdot 560 \cdot 320 + \overline{560} \cdot \overline{560} \cdot 320 + 560 \cdot 560 \cdot 10_i + \overline{560} \cdot \overline{560} \cdot 10_i \tag{15}$$

where the index $i = 1, 2$. No mass terms are included for the 10-plets and the 320–plet. We note that the heavy sector, i.e., $560 + \overline{560}$, gives us 4 doublets pairs and 5 triplet/anti-triplets pairs (which enter in the DT splitting mechanism) while the light sector consisting of $2 \times 10 + 320$ Higgs gives us 2D+2T from 2×10 and 3D+3T from 320 so that one has a total of 5D+5T from the light sector. Thus when the heavy and light sectors mix we are left with just one pair of light Higgs doublets while all other components of $560 + \overline{560}$ and of $2 \times 10 + 320$ will be super–heavy.

Now we show how all exotics from the 320 become massive in this model. The light 320 has the following set of exotics: $\overline{40}(6)$, $45(2)$, $70(2)$, $40(-6)$, $\overline{45}(-2), \overline{70}(-2)$. The exotics will combine with the heavy fields in $560 + \overline{560}$ via the couplings of Eq. (15). Mass growth for the exotics occurs as follows.

$$\begin{aligned}
&< (560, R_v, -5) > \cdot (560, 40, -1) \cdot (320, \overline{40}, 6), \qquad < (\overline{560}, \overline{R}_v, 5) > \cdot (\overline{560}, \overline{40}, 1) \cdot (320, 40, -6),\\
&< (560, R_v, -5) > \cdot (560, \overline{70}, 3) \cdot (320, 70, 2), \qquad < (\overline{560}, \overline{R}_v, 5) > \cdot (\overline{560}, 70, -3) \cdot (320, \overline{70}, -2),\\
&< (560, R_v, -5) > \cdot (560, 45, 7) \cdot (320, \overline{45}, -2), \qquad < (\overline{560}, \overline{R}_v, 5) > \cdot (\overline{560}, \overline{45}, -7) \cdot (320, 45, 2),\\
&< (560, R_v, -5) > \cdot (560, \overline{45}, 3) \cdot (320, 45, 2), \qquad < (\overline{560}, \overline{R}_v, 5) > \cdot (\overline{560}, 45, -3) \cdot (320, \overline{45}, -2),
\end{aligned} \tag{16}$$

where $R_v = 1, 24, 75$ are the $SU(5)$ representations from the $560 + \overline{560}$ that develop VEVs. We see that there is a one to one pairing of the light exotics with the heavy ones which will make all the light exotics heavy. The remaining components are just the 5–plets and $\bar{5}$–plets in Eq. (14) which enter in the DT splitting realizing the missing partner mechanism. So we find that the only massless Higgs fields left after spontaneous breaking are a pair of Higgs doublets and all the rest become super–heavy and are removed from the low energy spectrum.

One might wonder if one of the two 10–plets in the light sector can be replaced by a $16 + \overline{16}$, which also contain one pair of doublets and one triplets. Although one can write higher dimensional terms of the type $560 \cdot 560 \cdot 16^2$ for mixing color triplets, the 16 must develop a VEV along its SM singlet direction for this purpose. Setting the F–term for 16 and 560 would show that one of the two VEVs should vanish, in which case the missing partner mechanism is not effective.

We note in passing that each of these models have $SU(5)$ couplings of type $10 \cdot 10 \cdot 5$ and $\overline{10} \cdot \overline{10} \cdot \bar{5}$. Thus a flipped symmetry breaking chain, i.e., $SO(10) \rightarrow SU(5)_F \times U(1) \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y$ appears possible.

The results of the analysis, both with $126 + \overline{126}$ and $560 + \overline{560}$ heavy fields are summarized in Table 1.

Model	Heavy Fields	Light Fields	Pairs of D and T in Heavy Fields	Pairs of D and T in Light Fields	Residual Set of Light Modes
(i)	$126 + \overline{126} + 210$	$2 \times 10 + 120$	(2D+3T)+(D+T)	(2D+2T)+(2D+2T)	1D
(ii)	$126 + \overline{126} + 45$	$10 + 120$	(2D+3T)	(D+T)+(2D+2T)	1D
(iii)	$126 + \overline{126}$	$10 + 120$	(2D+3T)	(D+T)+(2D+2T)	1D
(iv)	$560 + \overline{560}$	$1 \times 320 + 2 \times 10$	4D+5T	(3D+3T)+ (2D+2T)	1D

Table 1: Exhibition of Higgs doublet pairs (D) consisting of up–type and down–type Higgs doublets, and Higgs triplet/anti-triplet (T) pairs in the $SO(10)$ missing partner models discussed in this work. In case (iii) the $126 + \overline{126}$ heavy fields do not acquire super–heavy VEVs. Additional $210+16+\overline{16}$ fields, which do not mix with the $126+\overline{126}$, are utilized for $SO(10)$ symmetry breaking in this case.

5 560 and $\overline{560}$ as Constrained 2$^{\text{nd}}$ Rank Antisymmetric Tensor-Spinor Multiplets

In this section we build up the technology to deal with the $560 + \overline{560}$ multiplets quantitatively. These results will be applied in Sec. 6 to study the one step symmetry breaking of $SO(10)$ down to the SM, and in Sec. 7 for the computation of the doublet and the triplet Higgs mass matrices using $320 + 2 \times 10$ light Higgs sector. As mentioned in Sec. 4, for the analysis to follow, it is convenient to consider an $SU(5) \times U(1)_X$ decomposition of the 560 multiplet. Thus the 560 multiplet under $SU(5) \times U(1)_X$ is

$$\begin{aligned} 560 \;\; = \;\; & 1(-5)[\mathbf{U}] \; + \; \overline{5}(3)[\mathbf{U}_i] \; + \; \overline{10}(-9)[\mathbf{U}_{ij}] \; + \; 10(-1)[\mathbf{U}^{ij}] \; + \; 10(-1)[\widehat{\mathbf{U}}^{ij}] \; + \; 24(-5)[\mathbf{U}^i_j] \\ & + 40(-1)[\mathbf{U}^{ijk}_l] \; + \; 45(7)[\mathbf{U}^{ij}_k] \; + \; \overline{45}(3)[\mathbf{U}^k_{ij}] \; + \; \overline{50}(3)[\mathbf{U}^{lm}_{ijk}] \; + \; \overline{70}(3)[\mathbf{U}^k_{(S)ij}] \\ & + 75(-5)[\mathbf{U}^{ij}_{kl}] \; + \; 175(-1)[\mathbf{U}^m_{[ijk]l}], \end{aligned} \tag{17}$$

where, for example, $1(-5)$ means that it is a singlet of $SU(5)$ with a $U(1)$ quantum number of -5 and where quantities in the brackets represent the tensorial structure of each $SU(5)$ multiplet. The 560 multiplet of $SO(10)$ is a 16-plet spinor with two tensor indices and we represent this tensor-spinor by $|\Theta^{560}_{\mu\nu}>$, where $|\Theta^{560}_{\mu\nu}>$ is anti-symmetric in the indices $\mu\nu$ and satisfies the constraint $\Gamma_\mu|\Theta^{560}_{\mu\nu}>= 0$. Here μ, ν are the $SO(10)$ indices, i.e., $\mu, \nu = 1, 2, \cdots 10$, and Γ_μ are the $SO(10)$

gamma matrices which satisfy the Clifford algebra relation $\{\Gamma_\mu, \Gamma_\nu\} = 2\delta_{\mu\nu} I$. The irreducible tensor-spinor $|\Theta^{560}_{\mu\nu}>$ can be obtained from the reducible tensor-spinor $|\Theta^{720}_{\mu\nu}>$ by subtraction of the 160 plet given by $\Gamma_\mu|\Theta^{720}_{\mu\nu}>$.

To proceed further, a useful approach is in terms of an oscillator expansion where one defines Γ_μ in terms of $SU(5)$ oscillators as follows[11, 12]: $\Gamma_{2i} = (b_i + b_i^\dagger), \Gamma_{2i-1} = -i(b_i - b_i^\dagger), i = 1, \cdots, 5$ where $b_i, b_i^\dagger$ satisfy the algebra $\{b_i, b_j^\dagger\} = \delta_{ij}$, $\{b_i, b_j\} = 0$ and $\{b_i^\dagger, b_j^\dagger\} = 0$. In terms of $b_i, b_i^\dagger$ the 720 (=16×45) multiplet has the $SU(5)$ oscillator expansion: $|\Theta^{720}_{\mu\nu}>= |0> \theta_{\mu\nu} + \frac{1}{2} b_i^\dagger b_j^\dagger |0> \theta^{ij}_{\mu\nu} + \frac{1}{24}\epsilon^{ijklm} b_j^\dagger b_k^\dagger b_l^\dagger b_m^\dagger |0> \theta_{i\mu\nu}$, where as stated earlier $\mu, \nu, ...$ are $SO(10)$ indices, while $i, j, k, l, ...$ and $SU(5)$ indices.

In the decomposition of $SO(10)$ tensors into $SU(5)$ tensors it is convenient to first express the $SO(10)$ tensors in terms of a specific set of $SU(5)$ reducible tensors. These can then be further decomposed into irreducible $SU(5)$ tensors. The explicit computations of the $SO(10)$ couplings involving the 560 multiplet require in addition the techniques of the Basic Theorem developed in [13] where one expresses the interaction structure using a Wick expansion in terms of the $SU(5)$ oscillators.

We now explicitly state the expansion of the 560 dimensional tensor-spinor in its $SU(5)$ oscillator modes where we have implemented the constraint $\Gamma_\mu|\Theta^{560}_{\mu\nu}>= 0$. The result of a detailed analysis gives

$$\begin{aligned}
|\Theta^{560}_{c_x c_y}> &= |0> \mathbf{U}^{xy} + \frac{1}{2} b_i^\dagger b_j^\dagger |0> \Big[\epsilon^{ijklm} \mathbf{U}^{xy}_{klm} \\
&+ \frac{1}{96}\left(\epsilon^{ijkly}\mathbf{U}^{x}_{kl} - \epsilon^{ijklx}\mathbf{U}^{y}_{kl}\right) - \frac{1}{288}\epsilon^{ijkxy}\mathbf{U}_k\Big] + \frac{1}{24}\epsilon^{ijklm} b_j^\dagger b_k^\dagger b_l^\dagger b_m^\dagger |0> \mathbf{U}^{xy}_i,
\end{aligned} \tag{18}$$

$$\begin{aligned}
|\Theta^{560}_{\bar{c}_x \bar{c}_y}> &= |0> \mathbf{U}_{xy} + \frac{1}{2} b_i^\dagger b_j^\dagger |0> \Big[\mathbf{U}^{ij}_{xy} + \frac{1}{3}\left(\delta^i_y \mathbf{U}^j_x - \delta^i_x \mathbf{U}^j_y + \delta^j_x \mathbf{U}^i_y - \delta^j_y \mathbf{U}^i_x\right) \\
&+ \frac{1}{20}\left(\delta^i_y \delta^j_x - \delta^i_x \delta^j_y\right)\mathbf{U}\Big] + \frac{1}{24}\epsilon^{ijklm} b_j^\dagger b_k^\dagger b_l^\dagger b_m^\dagger |0> [\epsilon_{nopxy}\mathbf{U}^{nop}_i \\
&+ \epsilon_{inoxy}\widehat{\mathbf{U}}^{no}\Big],
\end{aligned} \tag{19}$$

$$\begin{aligned}
|\Theta^{560}_{c_x \bar{c}_y}> &= |0> \left[\mathbf{U}^x_y + \frac{1}{5}\delta^x_y \mathbf{U}\right] + \frac{1}{2} b_i^\dagger b_j^\dagger |0> \Big[\frac{1}{4}\left(\delta^i_y \mathbf{U}^{jx} - \delta^j_y \mathbf{U}^{ix}\right) \\
&- \frac{1}{72}\left(4\delta^x_y \widehat{\mathbf{U}}^{ij} - \delta^j_y \widehat{\mathbf{U}}^{ix} + \delta^i_y \widehat{\mathbf{U}}^{jx}\right) - \frac{1}{6}\mathbf{U}^{ijx}_y + \epsilon^{ijklm}\mathbf{U}^x_{[klm]y}\Big] \\
&+ \frac{1}{24}\epsilon^{ijklm} b_j^\dagger b_k^\dagger b_l^\dagger b_m^\dagger |0> \left[\frac{1}{24}\left(5\delta^x_y \mathbf{U}_i - \delta^x_i \mathbf{U}_y\right) - \frac{1}{2}\left(\mathbf{U}^x_{iy} + \mathbf{U}^x_{(S)iy}\right)\right].
\end{aligned} \tag{20}$$

$$|\overline{\Theta}^{\overline{560}}_{c_x c_y}> = b_1^\dagger b_2^\dagger b_3^\dagger b_4^\dagger b_5^\dagger |0> \overline{\mathbf{U}}^{xy} + \frac{1}{12}\epsilon^{ijklm} b_k^\dagger b_l^\dagger b_m^\dagger |0> \Big[\overline{\mathbf{U}}^{xy}_{ij} + \frac{1}{3}\left(\delta^y_i \overline{\mathbf{U}}^x_j - \delta^x_i \overline{\mathbf{U}}^y_j + \delta^x_j \overline{\mathbf{U}}^y_i\right.$$

$$-\delta^y_j \overline{\mathbf{U}}^x_i\Big) + \frac{1}{20}\left(\delta^y_i \delta^x_j - \delta^x_i \delta^y_j\right)\overline{\mathbf{U}}\Big] + b^\dagger_i|0> \left[\epsilon^{nopxy}\overline{\mathbf{U}}^i_{nop} + \epsilon^{inoxy}\widehat{\overline{\mathbf{U}}}_{no}\right], \tag{21}$$

$$\begin{aligned}|\overline{\Theta}^{\overline{560}}_{\bar{c}_x\bar{c}_y}> &= b^\dagger_1 b^\dagger_2 b^\dagger_3 b^\dagger_4 b^\dagger_5|0> \overline{\mathbf{U}}_{xy} + \frac{1}{12}\epsilon^{ijklm} b^\dagger_k b^\dagger_l b^\dagger_m|0> \Big[\epsilon_{ijpqr}\overline{\mathbf{U}}^{pqr}_{xy} \\ &+ \frac{1}{96}\left(\epsilon_{ijpqy}\overline{\mathbf{U}}^{pq}_x - \epsilon_{ijpqx}\overline{\mathbf{U}}^{pq}_y\right) - \frac{1}{288}\epsilon_{ijpxy}\overline{\mathbf{U}}^p\Big] + b^\dagger_i|0> \overline{\mathbf{U}}^i_{xy},\end{aligned} \tag{22}$$

$$\begin{aligned}|\overline{\Theta}^{\overline{560}}_{c_x\bar{c}_y}> &= b^\dagger_1 b^\dagger_2 b^\dagger_3 b^\dagger_4 b^\dagger_5|0> \left[\overline{\mathbf{U}}^x_y + \frac{1}{5}\delta^x_y\overline{\mathbf{U}}\right] + \frac{1}{12}\epsilon^{ijklm} b^\dagger_k b^\dagger_l b^\dagger_m|0> \Big[\frac{1}{4}\left(\delta^x_i\overline{\mathbf{U}}_{jy} - \delta^x_j\overline{\mathbf{U}}_{iy}\right) \\ &- \frac{1}{72}\left(4\delta^x_y\widehat{\overline{\mathbf{U}}}_{ij} - \delta^x_j\widehat{\overline{\mathbf{U}}}_{iy} + \delta^x_i\widehat{\overline{\mathbf{U}}}_{jy}\right) - \frac{1}{6}\overline{\mathbf{U}}^x_{ijy} + \epsilon_{ijpqr}\overline{\mathbf{U}}^{[pqr]x}_y\Big] \\ &+ b^\dagger_i|0> \left[\frac{1}{24}\left(5\delta^x_y\overline{\mathbf{U}}^i - \delta^i_y\overline{\mathbf{U}}^x\right) - \frac{1}{2}\left(\overline{\mathbf{U}}^{ix}_y + \overline{\mathbf{U}}^{ix}_{(S)y}\right)\right].\end{aligned} \tag{23}$$

Eqs. (18-23) show how the various $SU(5)$ components given in Eq. (17) enter in the oscillator decomposition of the constrained 560 multiplet.

6 A More Unified $SO(10)$: A One Scale Breaking of $SO(10)$ GUT Symmetry with $560 + \overline{560}$

In this section we give an analysis of the breaking of the GUT symmetry when the 560 multiplet develops a VEV. As mentioned in Sec. 1 we have three possible candidates for the VEV formation in the 560 multiplet. In $SU(5)\times U(1)_X$ decomposition they are the elements $1(-5),\ 24(-5),\ 75(-5)$ in the 560 and similarly $\bar{1}(5),\ \overline{24}(5),\ \overline{75}(5)$ in $\overline{560}$. Suppose one breaks the GUT symmetry using just the 75 multiplet. In this case there will be a large number of unwanted pseudo-Goldstone bosons left over. To avoid this situation we will consider the case where $1(-5),\ 24(-5),\ 75(-5)$ and their counterparts all develop VEVs. Indeed one finds that the VEV growth for all the three fields is automatic unless one does an extreme fine tuning. Thus, in the analysis given below we allow all the three multiplets to develop VEVs and look for spontaneous breaking for this general case. (See also the Appendix).

In the analysis of spontaneous breaking we will use a combination of a bilinear term $560.\overline{560}$ and a quartic term $(560.\overline{560})^2$ to develop VEVs. The bilinear term necessary for the GUT symmetry breaking is simply the mass term for the $560/\overline{560}$ multiplet. To generate the necessary quartic coupling involving 560 and $\overline{560}$ multiplets, we start with the simplest possible contraction of 560 and $\overline{560}$ with the 45-plet of $SO(10)$, Φ. The relevant superpotential then takes the form

$$\mathsf{W} = \frac{1}{2!}M_{45}\Phi^{45}_{\mu\nu}\Phi^{45}_{\mu\nu} + \lambda_{45} < \Theta^{560*}_{\mu\nu}|B|\overline{\Theta}^{\overline{560}}_{\nu\sigma} > \Phi^{45}_{\mu\sigma} + M_{560} < \Theta^{560*}_{\mu\nu}|B|\overline{\Theta}^{\overline{560}}_{\mu\nu} > . \tag{24}$$

The first term in W represents the mass term for the 45-dimensional anti-symmetric tensor and the B entering Eq. (24) is the $SO(10)$ charge conjugation operator so that $B = -\imath\prod_{i=1}^{5}\left(b_i - b^\dagger_i\right)$.

Integrating out the 45-plet and giving the following VEVs to the 1-plets, the 24-plets, and the 75-plets of $SU(5)$ contained in the 560 and $\overline{560}$,

$$\begin{pmatrix} <\mathbf{U}> \\ <\overline{\mathbf{U}}> \end{pmatrix} \equiv \begin{pmatrix} \mathbf{S}_{(1)} \\ \overline{\mathbf{S}}_{(1)} \end{pmatrix}, \qquad \begin{pmatrix} <\mathbf{U}^{\alpha}_{\beta}> \\ <\overline{\mathbf{U}}^{\alpha}_{\beta}> \end{pmatrix} = \frac{1}{3}\delta^{\alpha}_{\beta} \begin{pmatrix} \mathbf{S}_{(24)} \\ \overline{\mathbf{S}}_{(24)} \end{pmatrix},$$

$$\begin{pmatrix} <\mathbf{U}^{a}_{b}> \\ <\overline{\mathbf{U}}^{a}_{b}> \end{pmatrix} = -\frac{1}{2}\delta^{a}_{b} \begin{pmatrix} \mathbf{S}_{(24)} \\ \overline{\mathbf{S}}_{(24)} \end{pmatrix}, \qquad \begin{pmatrix} <\mathbf{U}^{\alpha\beta}_{\gamma\sigma}> \\ <\overline{\mathbf{U}}^{\alpha\beta}_{\gamma\sigma}> \end{pmatrix} = \frac{1}{6}\left(\delta^{\alpha}_{\gamma}\delta^{\beta}_{\sigma} - \delta^{\alpha}_{\sigma}\delta^{\beta}_{\gamma}\right) \begin{pmatrix} \mathbf{S}_{(75)} \\ \overline{\mathbf{S}}_{(75)} \end{pmatrix}$$

$$\begin{pmatrix} <\mathbf{U}^{ab}_{cd}> \\ <\overline{\mathbf{U}}^{ab}_{cd}> \end{pmatrix} = \frac{1}{2}\left(\delta^{a}_{c}\delta^{b}_{d} - \delta^{a}_{d}\delta^{b}_{c}\right) \begin{pmatrix} \mathbf{S}_{(75)} \\ \overline{\mathbf{S}}_{(75)} \end{pmatrix}, \qquad \begin{pmatrix} <\mathbf{U}^{\alpha a}_{\beta b}> \\ <\overline{\mathbf{U}}^{\alpha a}_{\beta b}> \end{pmatrix} = -\frac{1}{6}\delta^{a}_{b}\delta^{\alpha}_{\beta} \begin{pmatrix} \mathbf{S}_{(75)} \\ \overline{\mathbf{S}}_{(75)} \end{pmatrix}, \tag{25}$$

we get,

$$\begin{aligned}
\mathsf{W} \;=\; & \frac{\lambda_{45}^2}{4M_{45}}\Bigg[-\left(\frac{7}{27}\right)\mathbf{S}^2_{(75)}\overline{\mathbf{S}}^2_{(75)} - \left(\frac{10}{81}\right)\mathbf{S}^2_{(75)}\overline{\mathbf{S}}_{(75)}\overline{\mathbf{S}}_{(24)} - \left(\frac{10}{81}\right)\mathbf{S}_{(75)}\overline{\mathbf{S}}^2_{(75)}\mathbf{S}_{(24)} \\
& -\left(\frac{349}{972}\right)\mathbf{S}_{(75)}\overline{\mathbf{S}}_{(75)}\mathbf{S}_{(24)}\overline{\mathbf{S}}_{(24)} + \left(\frac{7}{720}\right)\mathbf{S}_{(75)}\overline{\mathbf{S}}_{(75)}\mathbf{S}_{(24)}\overline{\mathbf{S}}_{(1)} + \left(\frac{7}{720}\right)\mathbf{S}_{(75)}\overline{\mathbf{S}}_{(75)}\overline{\mathbf{S}}_{(24)}\mathbf{S}_{(1)} \\
& -\left(\frac{1}{50}\right)\mathbf{S}_{(75)}\overline{\mathbf{S}}_{(75)}\mathbf{S}_{(1)}\overline{\mathbf{S}}_{(1)} - \left(\frac{125}{1944}\right)\overline{\mathbf{S}}^2_{(75)}\mathbf{S}^2_{(24)} - \left(\frac{125}{1944}\right)\mathbf{S}^2_{(75)}\overline{\mathbf{S}}^2_{(24)} \\
& +\left(\frac{5}{216}\right)\mathbf{S}_{(75)}\overline{\mathbf{S}}_{(24)}\mathbf{S}_{(24)}\overline{\mathbf{S}}_{(1)} + \left(\frac{5}{216}\right)\mathbf{S}_{(75)}\overline{\mathbf{S}}^2_{(24)}\mathbf{S}_{(1)} + \left(\frac{5}{216}\right)\overline{\mathbf{S}}_{(75)}\mathbf{S}^2_{(24)}\overline{\mathbf{S}}_{(1)} \\
& +\left(\frac{5}{216}\right)\overline{\mathbf{S}}_{(75)}\mathbf{S}_{(24)}\overline{\mathbf{S}}_{(24)}\mathbf{S}_{(1)} - \left(\frac{73}{2916}\right)\mathbf{S}_{(75)}\overline{\mathbf{S}}^2_{(24)}\mathbf{S}_{(24)} - \left(\frac{73}{2916}\right)\overline{\mathbf{S}}_{(75)}\mathbf{S}^2_{(24)}\overline{\mathbf{S}}_{(24)} \\
& +\left(\frac{1015}{17496}\right)\overline{\mathbf{S}}^2_{(24)}\mathbf{S}^2_{(24)} + \left(\frac{1}{648}\right)\mathbf{S}^2_{(24)}\overline{\mathbf{S}}_{(24)}\overline{\mathbf{S}}_{(1)} + \left(\frac{1}{648}\right)\overline{\mathbf{S}}^2_{(24)}\mathbf{S}_{(24)}\mathbf{S}_{(1)} \\
& -\left(\frac{1}{480}\right)\mathbf{S}^2_{(24)}\overline{\mathbf{S}}^2_{(1)} - \left(\frac{1}{480}\right)\overline{\mathbf{S}}^2_{(24)}\mathbf{S}^2_{(1)} + \left(\frac{1}{144}\right)\overline{\mathbf{S}}_{(24)}\mathbf{S}_{(24)}\mathbf{S}_{(1)}\overline{\mathbf{S}}_{(1)} \\
& +\left(\frac{1}{500}\right)\overline{\mathbf{S}}^2_{(1)}\mathbf{S}^2_{(1)}\Bigg] + \imath M_{560}\left[-\overline{\mathbf{S}}_{(75)}\mathbf{S}_{(75)} + \left(\frac{10}{9}\right)\overline{\mathbf{S}}_{(24)}\mathbf{S}_{(24)} + \left(\frac{7}{20}\right)\overline{\mathbf{S}}_{(1)}\mathbf{S}_{(1)}\right].
\end{aligned} \tag{26}$$

Here, for example, the 24–plet of $SU(5)$ contained in the 560–plet of $SO(10)$ has the Standard Model singlet denoted by $\mathbf{S}_{(24)}$, while, the 75–plet of $SU(5)$ contained in the $\overline{560}$–plet of $SO(10)$ has the Standard Model singlet denoted by $\overline{\mathbf{S}}_{(75)}$, etc..

Using Eq. (26) we now look for solutions for which

$$\frac{\partial \mathsf{W}}{\partial \mathbf{S}_{(75)}} = 0, \quad \frac{\partial \mathsf{W}}{\partial \overline{\mathbf{S}}_{(75)}} = 0, \quad \frac{\partial \mathsf{W}}{\partial \mathbf{S}_{(24)}} = 0, \quad \frac{\partial \mathsf{W}}{\partial \overline{\mathbf{S}}_{(24)}} = 0, \quad \frac{\partial \mathsf{W}}{\partial \mathbf{S}_{(1)}} = 0, \quad \frac{\partial \mathsf{W}}{\partial \overline{\mathbf{S}}_{(1)}} = 0,$$

are satisfied simultaneously. In general there are six VEVs to be determined. Since these are coupled cubic equations in six parameters it is not possible to solve these equations analytically. Further, the number of allowed solutions is very large. Thus the solutions to the VEV equations were carried out numerically on Mathematica. Since the number of allowed solutions is rather large we display here only a few sample cases. Thus, for the case when the VEVs of $1(-5)$ and $\overline{1}(5)$ are taken equal, and similarly for $24(-5)$ and $\overline{24}(5)$, and for $75(-5)$ and $\overline{75}(5)$, we display in Table 4

Table 2: $\mathbf{S}_{(1)} = \overline{\mathbf{S}}_{(1)}$, $\mathbf{S}_{(24)} = \overline{\mathbf{S}}_{(24)}$, $\mathbf{S}_{(75)} = \overline{\mathbf{S}}_{(75)}$

$M_{45} \cdot M_{560}$ (GeV^2)	$\mathbf{S}_{(1)}$ (GeV)	$\mathbf{S}_{(24)}$ (GeV)	$\mathbf{S}_{(75)}$ (GeV)
10^{30}	$(-1.2 + \imath 1.4) \times 10^{16}$	$(27 + \imath 5.7) \times 10^{14}$	$(6.4 + \imath 25) \times 10^{14}$
	$(-5.4 - \imath 5.4) \times 10^{12}$	$(-2.5 + \imath 2.5) \times 10^{14}$	$(2 - \imath 2) \times 10^{15}$
10^{31}	$(4.4 - \imath 3.8) \times 10^{16}$	$(1.8 + \imath 8.4) \times 10^{15}$	$(7.8 + \imath 2) \times 10^{15}$
	$(-1.7 + \imath 1.7) \times 10^{13}$	$(-7.9 + \imath 7.9) \times 10^{14}$	$(6.5 - \imath 6.5) \times 10^{15}$
10^{32}	$(1.2 - \imath 1.4) \times 10^{17}$	$(-27 - \imath 5.7) \times 10^{15}$	$(-6.4 - \imath 25) \times 10^{15}$
	$(-5.4 + \imath 5.4) \times 10^{13}$	$(-2.5 + \imath 2.5) \times 10^{15}$	$(2 - \imath 2) \times 10^{16}$
10^{33}	$(-3.8 + \imath 4.4) \times 10^{17}$	$(8.4 + \imath 1.8) \times 10^{16}$	$(2 + \imath 7.9) \times 10^{16}$
	$(1.7 - \imath 1.7) \times 10^{14}$	$(7.9 - \imath 7.9) \times 10^{15}$	$(-6.5 + \imath 6.5) \times 10^{16}$
10^{34}	$(1.4 - \imath 1.2) \times 10^{18}$	$(5.7 + \imath 27) \times 10^{16}$	$(25 + \imath 6.4) \times 10^{16}$
	$(-5.4 + \imath 5.4) \times 10^{14}$	$(-2.5 + \imath 2.5) \times 10^{16}$	$(2 - \imath 2) \times 10^{17}$

Table 2: Numerical estimates of the VEVs of the singlet, the 24–plet and the 75–plet fields in the $560 + \overline{560}$ multiplets in the spontaneous breaking of the $SO(10)$ gauge symmetry at the GUT scale.

a set of 10 solutions, two for each value of $M_{45} \cdot M_{560}$. We display one such solution below

$$M_{45} \cdot M_{560} = 10^{32} \text{ GeV}^2,$$
$$\mathbf{S}_{(1)} = (1.2 - \imath 1.4) \times 10^{17},\ \mathbf{S}_{(24)} = (-2.7 - \imath .57) \times 10^{16},\ \mathbf{S}_{(75)} = (-.64 - \imath 2.5) \times 10^{16}, \tag{27}$$

where the VEVs are all in the unit of GeV. The VEVs of Eq. (27) are close to the conventional unification scale of 2×10^{16} GeV. We note that in determining the VEVs only the product of $M_{45} \cdot M_{560}$ enters.

7 Higgs Doublets and Higgs Triplet Mass Matrices in $560 + \overline{560} + (2 \times 10 + 320)$ Model

In Table 3 we exhibit in detail the mass generation for the Higgs doublets. Here the entries $\mathsf{d}_1 - \mathsf{d}_{19}$ arise from the mixings of the 560 multiplet with 320 while the entries $\mathsf{d}_{20} - \mathsf{d}_{31}$ arise from mixings with the Higgs 10–plets $10_1,\ 10_2$. The entries $\mathsf{d}_1 - \mathsf{d}_{31}$ are listed below.

$$\mathsf{d}_1 = \begin{pmatrix} <1_{560}(-5)> \\ <24_{560}(-5)> \end{pmatrix} \cdot \overline{5}_{560}(3) \cdot 5_{320}(2), \qquad \mathsf{d}_2 = \begin{pmatrix} <24_{560}(-5)> \\ <75_{560}(-5)> \end{pmatrix} \cdot \overline{45}_{560}(3) \cdot 5_{320}(2)$$

$$\mathsf{d}_3 = (<24_{560}(-5)>) \cdot \overline{70}_{560}(3) \cdot 5_{320}(2), \qquad \mathsf{d}_4 = \begin{pmatrix} <\overline{24}_{\overline{560}}(5)> \\ <\overline{75}_{\overline{560}}(5)> \end{pmatrix} \cdot \overline{45}_{\overline{560}}(-7) \cdot 5_{320}(2),$$

$$\mathsf{d}_5 = \begin{pmatrix} <\overline{1}_{\overline{560}}(5)> \\ <\overline{24}_{\overline{560}}(5)> \end{pmatrix} \cdot 5_{\overline{560}}(-3) \cdot \overline{5}_{320}(-2) \qquad \mathsf{d}_6 = \begin{pmatrix} <\overline{24}_{\overline{560}}(5)> \\ <\overline{75}_{\overline{560}}(5)> \end{pmatrix} \cdot 45_{\overline{560}}(-3) \cdot \overline{5}_{320}(-2)$$

$$\mathsf{d}_7 = \left(<\overline{24}_{\overline{560}}(5)>\right) \cdot 70_{\overline{560}}(-3) \cdot \overline{5}_{320}(-2), \qquad \mathsf{d}_8 = \begin{pmatrix} <\overline{1}_{\overline{560}}(5)> \\ <\overline{24}_{\overline{560}}(5)> \\ <\overline{75}_{\overline{560}}(5)> \end{pmatrix} \cdot \overline{40}_{\overline{560}}(1) \cdot 40_{320}(-6)$$

	5_{320}	$\overline{5}_{320}$	40_{320}	$\overline{40}_{320}$	45_{320}	$\overline{45}_{320}$	70_{320}	$\overline{70}_{320}$	$5_{1_{10_1}}$	$\overline{5}_{1_{10_1}}$	$5_{2_{10_2}}$	$\overline{5}_{2_{10_2}}$
$\overline{5}_{560}$	d_1				d_{10}				d_{24}		d_{30}	
40_{560}				d_9								
$\overline{45}_{560}$	d_2				d_{11}		d_{18}		d_{25}		d_{31}	
45_{560}						d_{14}		d_{21}		d_{27}		d_{33}
$\overline{70}_{560}$	d_3				d_{12}		d_{19}					
$5_{\overline{560}}$		d_5				d_{15}				d_{28}		d_{34}
$\overline{40}_{\overline{560}}$			d_8									
$45_{\overline{560}}$		d_6				d_{16}		d_{22}		d_{29}		d_{35}
$\overline{45}_{\overline{560}}$	d_4				d_{13}		d_{20}		d_{26}		d_{32}	
$70_{\overline{560}}$		d_7				d_{17}		d_{23}				

Table 3: A list of non-vanishing mass terms for the Higgs doublets in the $560 + \overline{560} + 320 + 2 \times 10$ missing partner model.

$$\mathsf{d}_9 = \begin{pmatrix} < 1_{560}(-5) > \\ < 24_{560}(-5) > \\ < 75_{560}(-5) > \end{pmatrix} \cdot 40_{560}(-1) \cdot \overline{40}_{320}(6), \qquad \mathsf{d}_{10} = \begin{pmatrix} < 24_{560}(-5) > \\ < 75_{560}(-5) > \end{pmatrix} \cdot \overline{5}_{560}(3) \cdot 45_{320}(2)$$

$$\mathsf{d}_{11} = \begin{pmatrix} < 1_{560}(-5) > \\ < 24_{560}(-5) > \\ < 75_{560}(-5) > \end{pmatrix} \cdot \overline{45}_{560}(3) \cdot 45_{320}(2), \qquad \mathsf{d}_{12} = \begin{pmatrix} < 24_{560}(-5) > \\ < 75_{560}(-5) > \end{pmatrix} \cdot \overline{70}_{560}(3) \cdot 45_{320}(2)$$

$$\mathsf{d}_{13} = \begin{pmatrix} < \overline{1}_{\overline{560}}(5) > \\ < \overline{24}_{\overline{560}}(5) > \\ < \overline{75}_{\overline{560}}(5) > \end{pmatrix} \cdot \overline{45}_{\overline{560}}(-7) \cdot 45_{320}(2), \qquad \mathsf{d}_{14} = \begin{pmatrix} < 1_{560}(-5) > \\ < 24_{560}(-5) > \\ < 75_{560}(-5) > \end{pmatrix} \cdot 45_{560}(7) \cdot \overline{45}_{320}(-2)$$

$$\mathsf{d}_{15} = \begin{pmatrix} < \overline{24}_{\overline{560}}(5) > \\ < \overline{75}_{\overline{560}}(5) > \end{pmatrix} \cdot 5_{\overline{560}}(-3) \cdot \overline{45}_{320}(-2), \qquad \mathsf{d}_{16} = \begin{pmatrix} < \overline{1}_{\overline{560}}(5) > \\ < \overline{24}_{\overline{560}}(5) > \\ < \overline{75}_{\overline{560}}(5) > \end{pmatrix} \cdot 45_{\overline{560}}(-3) \cdot \overline{45}_{320}(-2)$$

$$\mathsf{d}_{17} = \begin{pmatrix} < \overline{24}_{\overline{560}}(5) > \\ < \overline{75}_{\overline{560}}(5) > \end{pmatrix} \cdot 70_{\overline{560}}(-3) \cdot \overline{45}_{320}(-2), \qquad \mathsf{d}_{18} = \begin{pmatrix} < 24_{560}(-5) > \\ < 75_{560}(-5) > \end{pmatrix} \cdot \overline{45}_{560}(3) \cdot 70_{320}(2)$$

$$\mathsf{d}_{19} = \begin{pmatrix} < 1_{560}(-5) > \\ < 24_{560}(-5) > \\ < 75_{560}(-5) > \end{pmatrix} \cdot \overline{70}_{560}(3) \cdot 70_{320}(2), \qquad \mathsf{d}_{20} = \begin{pmatrix} < \overline{24}_{\overline{560}}(5) > \\ < \overline{75}_{\overline{560}}(5) > \end{pmatrix} \cdot \overline{45}_{\overline{560}}(-7) \cdot 70_{320}(2)$$

$$
\begin{aligned}
\mathsf{d}_{21} &= \begin{pmatrix} <24_{560}(-5)> \\ <75_{560}(-5)> \end{pmatrix} \cdot 45_{560}(7) \cdot 70_{320}(-2), &
\mathsf{d}_{22} &= \begin{pmatrix} <\overline{24}_{\overline{560}}(5)> \\ <\overline{75}_{\overline{560}}(5)> \end{pmatrix} \cdot 45_{\overline{560}}(-3) \cdot \overline{70}_{320}(-2) \\
\mathsf{d}_{23} &= \begin{pmatrix} <\overline{1}_{\overline{560}}(5)> \\ <\overline{24}_{\overline{560}}(5)> \\ <\overline{75}_{\overline{560}}(5)> \end{pmatrix} \cdot 70_{\overline{560}}(-3) \cdot \overline{70}_{320}(-2), &
\mathsf{d}_{24} &= \begin{pmatrix} <1_{560}(-5)> \\ <24_{560}(-5)> \end{pmatrix} \cdot \overline{5}_{560}(3) \cdot 5_{1_{10_1}}(2) \\
\mathsf{d}_{25} &= \begin{pmatrix} <24_{560}(-5)> \\ <75_{560}(-5)> \end{pmatrix} \cdot \overline{45}_{560}(3) \cdot 5_{1_{10_1}}(2), &
\mathsf{d}_{26} &= \begin{pmatrix} <\overline{24}_{\overline{560}}(5)> \\ <\overline{75}_{\overline{560}}(5)> \end{pmatrix} \cdot \overline{45}_{\overline{560}}(-7) \cdot 5_{1_{10_1}}(2) \\
\mathsf{d}_{27} &= \begin{pmatrix} <24_{560}(-5)> \\ <75_{560}(-5)> \end{pmatrix} \cdot 45_{560}(7) \cdot \overline{5}_{1_{10_1}}(-2), &
\mathsf{d}_{28} &= \begin{pmatrix} <\overline{1}_{\overline{560}}(5)> \\ <\overline{24}_{\overline{560}}(5)> \end{pmatrix} \cdot 5_{\overline{560}}(-3) \cdot \overline{5}_{1_{10_1}}(-2) \\
\mathsf{d}_{29} &= \begin{pmatrix} <\overline{24}_{\overline{560}}(5)> \\ <\overline{75}_{\overline{560}}(5)> \end{pmatrix} \cdot 45_{\overline{560}}(-3) \cdot \overline{5}_{1_{10_1}}(-2), &
\mathsf{d}_{30} &= \begin{pmatrix} <1_{560}(-5)> \\ <24_{560}(-5)> \end{pmatrix} \cdot \overline{5}_{560}(3) \cdot 5_{2_{10_2}}(2) \\
\mathsf{d}_{31} &= \begin{pmatrix} <24_{560}(-5)> \\ <75_{560}(-5)> \end{pmatrix} \cdot \overline{45}_{560}(3) \cdot 5_{2_{10_2}}(2), &
\mathsf{d}_{32} &= \begin{pmatrix} <\overline{24}_{\overline{560}}(5)> \\ <\overline{75}_{\overline{560}}(5)> \end{pmatrix} \cdot \overline{45}_{\overline{560}}(-7) \cdot 5_{2_{10_2}}(2) \\
\mathsf{d}_{33} &= \begin{pmatrix} <24_{560}(-5)> \\ <75_{560}(-5)> \end{pmatrix} \cdot 45_{560}(7) \cdot \overline{5}_{2_{10_2}}(-2), &
\mathsf{d}_{34} &= \begin{pmatrix} <\overline{1}_{\overline{560}}(5)> \\ <\overline{24}_{\overline{560}}(5)> \end{pmatrix} \cdot 5_{\overline{560}}(-3) \cdot \overline{5}_{2_{10_2}}(-2) \\
\mathsf{d}_{35} &= \begin{pmatrix} <\overline{24}_{\overline{560}}(5)> \\ <\overline{75}_{\overline{560}}(5)> \end{pmatrix} \cdot 45_{\overline{560}}(-3) \cdot \overline{5}_{2_{10_2}}(-2). & &
\end{aligned}
\tag{28}
$$

The mass matrix arising from Table 3 leads to one pair of massless Higgs doublets while all the other Higgs doublets become heavy.

The triplet/anti-triplet mass matrix is given in Table 4. Here $\mathsf{t}_1 - \mathsf{t}_{35}$ are defined similar to $\mathsf{d}_1 - \mathsf{d}_{31}$, except that one is extracting the Higgs triplets/anti-triplets couplings rather than the Higgs doublet couplings. The new contributions to the Higgs triplet/anti-triplets mass matrix arise from the terms $\mathsf{T}_1 - \mathsf{T}_8$ which are given by

$$
\begin{aligned}
\mathsf{T}_1 &=< 75_{560}(-5) > \cdot \overline{50}_{560}(3) \cdot 5_{320}(2), &
\mathsf{T}_2 &=< \overline{75}_{\overline{560}}(5) \cdot 50_{\overline{560}}(-3) \cdot \overline{5}_{320}(-2) \\
\mathsf{T}_3 &= \begin{pmatrix} <24_{560}(-5)> \\ <75_{560}(-5)> \end{pmatrix} \cdot \overline{50}_{560}(3) \cdot 45_{320}(2), &
\mathsf{T}_4 &= \begin{pmatrix} <\overline{24}_{\overline{560}}(5)> \\ <\overline{75}_{\overline{560}}(5)> \end{pmatrix} \cdot 50_{\overline{560}}(-3) \cdot \overline{45}_{320}(-2) \\
\mathsf{T}_5 &=< 75_{560}(-5) > \cdot \overline{50}_{560}(3) \cdot 5_{1_{10_1}}(2), &
\mathsf{T}_6 &=< \overline{75}_{\overline{560}}(5) \cdot 50_{\overline{560}}(-3) \cdot \overline{5}_{1_{10_1}}(-2) \\
\mathsf{T}_7 &=< 75_{560}(-5) > \cdot \overline{50}_{560}(3) \cdot 5_{2_{10_2}}(2), &
\mathsf{T}_8 &=< \overline{75}_{\overline{560}}(5) \cdot 50_{\overline{560}}(-3) \cdot \overline{5}_{2_{10_2}}(-2).
\end{aligned}
$$

The triplet/anti-triplet mass matrix arising from the entries of Table 4 is rank 10 and thus all the Higgs triplets/anti-triplets become super heavy and are removed from the low energy spectrum of the theory. Further, as shown in Sec. 4 all the remaining components of the 320 multiplet become massive. Thus at the end we are left with only one light pair of $SU(2)_L$ Higgs doublets and all the remaining Higgs fields will be super–heavy.

8 Fermion Masses, $b - t - \tau$ Unification and Proton Decay

The matter fields as usual will reside in three generations of 16–plets and these will have cubic couplings with the light Higgs doublets. Typically in models discussed in Table 1 the light Higgs

	5_{320}	$\overline{5}_{320}$	40_{320}	$\overline{40}_{320}$	45_{320}	$\overline{45}_{320}$	70_{320}	$\overline{70}_{320}$	$5_{1_{10_1}}$	$\overline{5}_{1_{10_1}}$	$5_{2_{10_2}}$	$\overline{5}_{2_{10_2}}$
$\overline{5}_{560}$	t_1				t_{10}				t_{24}		t_{30}	
40_{560}				t_9								
$\overline{45}_{560}$	t_2				t_{11}		t_{18}		t_{25}		t_{31}	
45_{560}						t_{14}		t_{21}		t_{27}		t_{33}
$\overline{70}_{560}$	t_3				t_{12}		t_{19}					
$5_{\overline{560}}$		t_5				t_{15}				t_{28}		t_{34}
$\overline{40}_{\overline{560}}$			t_8									
$45_{\overline{560}}$		t_6				t_{16}		t_{22}		t_{29}		t_{35}
$\overline{45}_{\overline{560}}$	t_4				t_{13}		t_{20}		t_{26}		t_{32}	
$70_{\overline{560}}$		t_7				t_{17}		t_{23}				
$\overline{50}_{560}$	T_1				T_3				T_5		T_7	
$50_{\overline{560}}$		T_2				T_4				T_6		T_8

Table 4: A list of non-vanishing mass terms for the Higgs triplets/anti-triplets in the $560 + \overline{560} + 320 + 2 \times 10$ missing partner model.

doublets will be linear combination of doublets in 10–plets and in 120–plets of Higgs fields. In addition one can produce non-minimal models by adding an arbitrary (equal) number of light and heavy generations of 10's 120's and $126 + \overline{126}$'s. As an example, one may add a light and a heavy $126' + \overline{126'}$ with a mixing between the two terms, i.e., one adds additional non-minimal terms as follows

$$W_{\rm non-min} = M\, 126'_h.\overline{126'}_h + M'\,(126'_h.\overline{126'}_l + 126'_l.\overline{126'}_h). \tag{29}$$

Here $126'_h$ is the heavy Higgs multiplets with mass M while $126'_l$ is the light Higgs multiplet which has no mass term but it mixes with the heavy multiplet $126'$. In this case the light Higgs will in general be a linear combination of $10 + 120 + 126$. The addition of these combinations of light and heavy fields will not disturb the overall count of the light pair of doublets. Using the above technique fully realistic models for fermion masses are possible in all cases discussed in Table 1.

Specifically, consider models (ii) and (iii) listed in Table 1. The MSSM Higgs fields (H_u, H_d) have components in a 10 and a 120 in these cases. If we add to this model a heavy and light $126 + \overline{126}$ as shown in Eq. (29), (H_u, H_d) will also have components from the light $126 + \overline{126}$. Such models can generate fully consistent quark and lepton masses, and also large neutrino mixing angles. The case where the Yukawa couplings to the light 120 is small would be a special case of a minimal $SO(10)$ model that has been widely studied [14]. This model actually predicts a relatively large value of $\sin^2 2\theta_{13} \approx 0.085$, which is compatible with the recent results from T2K [15] and MINOS [16] experiments. While the models presented here do not predict the value of θ_{13}, this special case suggests that they can certainly accommodate such large values for this neutrino mixing angle.

Next we consider briefly the fermion masses and mixings in the $560+ \overline{56} + 10_1 + 10_2 + 320$ model. Since the 320 does not couple to 16_i matter multiplets, the symmetric coupling matrices of 10_i $(i = 1, 2)$ would lead to the GUT scale degeneracy of down–type quarks and charged leptons. One could utilize the light–heavy $126+\overline{126}$ pair as in Eq. (29) to correct these relations. The heavy $\overline{126}$ can be used to generate large Majorana masss for the right–handed neutrinos. Alternatively, one could use the non-renormalizable coupling $16_i\ 16_j\ \overline{560}\ \overline{560}$ couplings. With two 10–plets and a $\overline{126}$–plet of light fields, there are three symmetric Yukawa coupling matrices, giving enough freedom to correct all the wrong mass relations that would arise in the absence of the $\overline{126}$–plet. Thus one can fit all the observables and specifically one has the possibility of producing a large θ_{13} in the neutrino mixing matrix.

We further note that since H_u, H_d are now linear combinations of several fields, the couplings of H_u and H_d are not necessarily equal at the unification scale. The prediction of $b - \tau$ unification still holds since b and τ reside in the same $SU(5)$ multiplet and their masses arise from the same H_d. However, the usual $SO(10)$ prediction that $b - t - \tau$ unification which requires large $\tan\beta$ [17] need not hold in this model for the reasons given above, i.e., that the couplings of H_u and of H_d are not necessarily equal at the unification scale. Thus in such models it is possible to achieve a $b - t - \tau$ unification without the necessity of a large $\tan\beta$. Further, the Higgs triplet couplings in such models will as usual produce baryon and lepton number violating dimension five operators (For a review see [18]). However, since the Higgs triplets/anti-triplets are linear combinations of several mass eigenstates it appears possible to suppress them via the cancellation mechanism [19], i.e., via an appropriate choice of the mixing angles that enter in the linear combinations. Detailed analyses of these issues are outside the scope of this work.

9 Conclusion

Models based on $SO(10)$ are desirable for a variety of reasons and continue to attract attention (For recents work see [20] and the references therein). In this paper we have analyzed the missing

partner mechanism anchored by the $126 + \overline{126}$ Higgs fields and found three consistent models. We have also proposed and developed a new class of missing partner $SO(10)$ models anchored by the $560 + \overline{560}$ multiplets. Here, for the light states an essentially unique possibility is found, consisting of $\{2 \times 10 + 320\}$ multiplets. This class of models allow for spontaneous breaking of the $SO(10)$ gauge group to the Standard Model gauge group in one step at the unification scale, in contrast to the conventional $SO(10)$ models where one needs two scales, i.e., one scale to break the rank of the group, and the second scale to reduce the gauge symmetry down to $SU(3)_C \times SU(2)_L \times U(1)_Y$. Further, the models lead to a natural doublet-triplet splitting via the missing partner mechanism where all exotic fields become heavy leaving only a pair of light Higgs doublets. The couplings of light Higgs doublets H_d and H_u are not necessarily equal at the unification scale since the light Higgses are linear combination of various multiplets. Because of this the low energy phenomenology of this model is different from a class of $SO(10)$ models where the MSSM parameter $\tan\beta$ takes a large value approximately equal to m_t/m_b. It is found that fully realistic models emerge for some cases, while others can be made realistic by addition of vector–like representations. It appears possible to suppress proton decay via dimension five operators but this analysis along with other phenomenological issues requires a further investigation.

We note that one of the important issues concerns the gauge coupling unification scale M_G. As in well known gauge coupling unification is affected by heavy thresholds. A possibility of cancelations in the threshold effects among different fragments of $560 + \overline{560}$ exists. However, this requires a detailed computation of all the heavy thresholds in the theory which is outside the scope of this work.

Finally, we note that the theory is not asymptotically free above the unification scale M_G because of the large number of degrees of freedom. However, the physics beyond the unification scale is largely unknown because one is entering the domain where gravity becomes strong. This is specifically the case when one has a large number of degrees of freedom N since as pointed out recently [21] the effective fundamental scale here is reduced by a factor $\sqrt{N}$ which thus lies close to the scale M_G. The above implies that non-renormalizable interactions due to the closeness of the fundamental scale could be large and must be included above M_G, which would redefine the theory above this scale. More specifically, above the scale M_G we should only work with the UV complete theory in this case.

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Appendix: Further Details of spontaneous breaking with $560 + \overline{560}$ Higgs and Absence of Goldstones

We discuss here some further aspects of spontaneous breaking with the $560+\overline{560}$ of Higgs fields. The simplest superpotential that would induce symmetry breaking $SO(10) \to SU(3)_C \times SU(2)_L \times U(1)_Y$ is of the form

$$W = M\ 560 \cdot \overline{560} + \frac{1}{M'}(560 \cdot \overline{560})_r \cdot (560 \cdot \overline{560})_r \tag{30}$$

where r stands for the representation by which 560 contracts. Let us suppose the contraction is trivial, i.e., r is a singlet of $SO(10)$. In this case the Lagrangian will have an $SU(560)$ global symmetry and a VEV formation of any singlet of 560 will lead to 559 Goldstone bosons. To avoid this situation one must consider a non-trivial contraction and the simplest such contraction is when $r = 45$, i.e., one considers the case $(560 \cdot \overline{560})_{45} \cdot (560 \cdot \overline{560})_{45}$. For the spontaneous symmetry breaking analysis it is useful to keep the 45–plet in the spectrum while analyzing the minimum of the potential, and then take the mass of the 45 to be very large to remove it from the spectrum. The superpotential couplings will then have a mass term for 45, a mass term M for the 560, and in addition will have the following cubic coupling:

$$\begin{aligned} 560 \cdot \overline{560} \cdot 45 &= 1 \cdot \overline{1} \cdot \widehat{1} + 1 \cdot \overline{24} \cdot \widehat{24} + 24 \cdot \overline{1} \cdot \widehat{24} + 24 \cdot \overline{24} \cdot \widehat{24} \\ &+ 24 \cdot \overline{75} \cdot \widehat{24} + 75 \cdot \overline{24} \cdot \widehat{24} + 75 \cdot \overline{75} \cdot \widehat{24} \end{aligned} \tag{31}$$

Here the hatted components are from 45, the bars arise from the $\overline{560}$, and have opposite $U(1)_X$ charges compared to the un-barred components from 560. Only the components which contain the SM singlets are shown in Eq. (31). Now a VEV formation for the $SU(5)$ singlet alone will not break the gauge symmetry completely down to the SM gauge group and one would need in addition the VEV formation for either the 24–plet or the 75–plet. However, we will show that the VEV formation of either the 24–plet or the 75 plet will lead to VEV formation for all the rest.

Let us consider the case when say the 24 and $\overline{24}$–plets get VEVs. Then from the 4th term on the right hand side of Eq. (31) one finds that $\widehat{24}$ will get a VEV since it is linear in $\widehat{24}$, and then from the 5th and 6th terms on the right hand side of Eq. (31) one finds that 75 and $\overline{75}$ will get VEVs. Further from the second and the third terms on the right hand side of Eq. (31) one finds that 1 and $\overline{1}$ will get VEVs. Thus one arrives at the result that if 24–plet gets a VEV, then $1, \overline{1}, 75, \overline{75}$ all get VEVS. A similar argument shows that if 75 and $\overline{75}$ get VEVs then the rest, i.e., $1, \overline{1}, 24, \overline{24}$ all get VEVs. The above argument can be repeated if $r = 210$ in Eq. (30). Specifically the expansion of the cubic coupling $560 \cdot \overline{560} \cdot 210$ gives the SM singlets as follows

$$\begin{aligned} 560 \cdot \overline{560} \cdot 210 &= 1 \cdot \overline{1} \cdot \widehat{1} + 24 \cdot \overline{24} \cdot \widehat{1} + 75 \cdot \overline{75} \cdot \widehat{1} + 1 \cdot \overline{24} \cdot \widehat{24} + 24 \cdot \overline{1} \cdot \widehat{24} \\ &+ 24 \cdot \overline{24} \cdot \widehat{24} + 24 \cdot \overline{75} \cdot \widehat{24} + 75 \cdot \overline{24} \cdot \widehat{24} + 75 \cdot \overline{75} \cdot \widehat{24} + 1 \cdot \overline{75} \cdot \widehat{75} + 24 \cdot \overline{24} \cdot \widehat{75} \\ &+ 24 \cdot \overline{75} \cdot \widehat{75} + 75 \cdot \overline{24} \cdot \widehat{75} + 75 \cdot \overline{75} \cdot \widehat{75}. \end{aligned} \tag{32}$$

where the hatted fields are from the 210. Eq. (32) is very similar to Eq.(31) except that this time we also have $\widehat{75}$ in addition to $\widehat{1}$, $\widehat{24}$. However, the analysis in this case gives exactly the same result, i.e., if either the $24, \overline{24}$ or the $75, \overline{75}$ get VEVs, then all the rest of the fields get VEVs as in the $r = 45$ case.

The quartic term in Eq. (30) can also arise by the contractions $(560 \cdot 560)_{\overline{126}}$, $(560 \cdot 560)_{120}$ or $(560 \cdot 560)_{10}$, which are all allowed. Among these, the 120 and 10 contractions will not generate quartic terms involving SM singlet, since these fields (10 and 120) do not contain SM singlets. The 126 contraction is a possibility. However, in this case, the symmetry breaks down to $SU(5)$ only, or else, many components of the 75-fragment of the 560 will remain massless. The superpotential in this case has mass terms for the 560 and for the 126, as well as two cubic terms: $(560 \cdot 560)_{\overline{126}}$ and $(\overline{560} \cdot \overline{560})_{126}$. The singlet components from the cubic terms are

$$\begin{aligned} 560 \cdot 560 \cdot \overline{126} &= 1 \cdot 1 \cdot \widehat{1} + 24 \cdot 24 \cdot \widehat{1} + 75 \cdot 75 \cdot \widehat{1} \\ \overline{560} \cdot \overline{560} \cdot 126 &= \overline{1} \cdot \overline{1} \cdot \widehat{\widehat{1}} + \overline{24} \cdot \overline{24} \cdot \widehat{\widehat{1}} + \overline{75} \cdot \overline{75} \cdot \widehat{\widehat{1}} \end{aligned} \tag{33}$$

where $\widehat{1}$ refers to the singlet fragment of $\overline{126}$ and $\widehat{\widehat{1}}$ is the singlet fragment from the 126. The mass terms are explicitly given by

$$W_{mass} = M(1 \cdot \overline{1} + 24 \cdot \overline{24} + 75 \cdot \overline{75}) + M_{126}\ \widehat{1} \cdot \widehat{\widehat{1}} \tag{34}$$

It is possible to obtain a minimum of Eqs. (33) and (34) with some of the VEVs zero. Thus, for example, one may have $1 = \overline{1} = 24 = \overline{24} = 0$ while $\widehat{1}, \widehat{\widehat{1}}, 75, \overline{75}$ all nonzero. However, if such a solution is chosen, one would end up with many components of $75 + \overline{75}$ being massless. The reason is that Eqs. (33)-(34) have an $O(75)$ symmetry, with both the 75 and $\overline{75}$ transforming as vectors. If the SM singlets in the 75 acquire VEVs, this global symmetry is broken down to $O(74)$, which generates many unwanted Goldstone bosons. On the other hand, if the singlets $(1, \overline{1}, \widehat{1}, \widehat{\widehat{1}})$ from 560 and 126 are taken to have non-zero VEV's, there is no such problem with Goldstones. However, in this case the unbroken symmetry is $SU(5)$. This proves that integrating out 126 can only result in $SO(10) \to SU(5)$ symmetry breaking. Next suppose we consider representations higher than 126 containing SM singlets that are in the contraction $(560 \cdot 560)$. This contraction has odd number of $SO(10)$ vector indices (odd number from the two spinor indices, and even number of vector indices, resulting in odd number of indices). Odd number of vector indices cannot contain 24 or 75 fragments; only possible SM singlet is in the 1 of $SU(5)$. Then the argument given for the 126 contraction will go through for higher representations as well and one will have the $SO(10) \to SU(5)$ symmetry breaking. The above analysis implies that in the analysis of spontaneous breaking one must consider non-trivial contractions, i.e., with r is Eq. (30) which is a non-singlet and specifically 45 or 210 and further the VEV growths for all the relevant fields, i.e., $1, 24, 75$ in the 560 multiplet must be taken into account for consistency.

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